Acoustic Attenuation in a Nonuniform Gas Containing Droplets

F. A. Lyman* and D.-M. Chen†
Syracuse University, Syracuse, N.Y.

A theoretical study of acoustic attenuation in a nonuniform gas-vapor-droplet mixture is presented. Condensation of vapor on the droplets causes temperature and density gradients along the direction of flow, and these gradients affect the attenuation in sound pressure level experienced by a plane acoustic wave propagating parallel to the flow. The general equations for unsteady one-dimensional flow of an inviscid gas, which contain source terms to model the mass, momentum, and entropy changes due to the droplets, are reduced to ordinary differential equations for the acoustic pressure and velocity fluctuations and for the mean flow variables. These equations are solved numerically, using explicit expressions for the source terms derived from consideration of the interaction between the droplets and the gas-vapor mixture. The results show that the attenuation in sound pressure level in a gas-vapor-droplet mixture with temperature and density gradients may be significantly different from that in a spatially uniform mixture.

Nomenclature

a = droplet radius
C_m = $nm_p/\bar{\rho}$, droplet mass fraction
c = speed of sound
c_ℓ = specific heat of liquid
c_p = specific heat of gas-vapor mixture at constant pressure
D = binary diffusion coefficient of vapor in gas
f = force per unit volume exerted on gas by droplets
H = $h_L/(R_v\bar{T}_p)$
h_L = latent heat of vaporization
I = $p'u'$, sound intensity
$Im(\ )$ = imaginary part of ( )
i = $\sqrt{-1}$
k = thermal conductivity of gas-vapor mixture
L = $\bar{u}\tau_{D0}/C_{m0}$, characteristic length
L_I = $10\log_{10}(I/I_{\rm ref})$, intensity level
L_P = $20\log_{10}(\sqrt{\overline{p'^2}}/p_{\rm ref})$, sound pressure level
L_V = $20\log_{10}(\sqrt{\overline{u'^2}}/u_{\rm ref})$, sound velocity level
M = $\bar{u}/\bar{c}$, Mach number of steady flow
m = rate of mass addition per unit volume
m_p = $(4\pi/3)\rho_\ell a^3$, droplet mass
N_{Le} = $k/(\rho c_p D) = \tau_D/\tau_T$, Lewis number
n = number of droplets per unit volume
P = dimensionless acoustic pressure, Eq. (38)
p = pressure
q = rate of heat addition per unit volume
R = gas constant
$Re(\ )$ = real part of ( )
r = ρ_v/ρ, vapor mass fraction
s = entropy per unit mass
T = absolute temperature
t = time
U = dimensionless acoustic velocity, Eq. (38)
u = flow velocity
W = molecular weight
x = distance
y_1,y_2,y_3,y_4 = $\bar{r}/\bar{r}_0$, $\bar{T}/\bar{T}_0$, $\bar{T}_p/\bar{T}_{p0}$, a/a_0, respectively
Z = p'/u', acoustic impedance
α = attenuation coefficient, dB/m
Γ_1,Γ_2 = coefficients defined by Eqs. (29a,b)
γ = ratio of specific heats of gas-vapor mixture
δ = arg(Z), phase angle
ζ = R_vH/c_ℓ
η = $(c_p/c_\ell)(\bar{T}/\bar{T}_p)H$
θ = $\bar{T}_p/\bar{T}$
κ = complex wave number
κ_0 = $\omega/\bar{c}_0$
λ = acoustic wavelength
μ = dynamic viscosity of gas-vapor mixture
ξ = x/L, dimensionless distance
Π_1,Π_2 = coefficients defined by Eqs. (30a,b)
ρ = density
σ' = $(p_v'/\bar{p}_v)-(p'/\bar{p})$
τ = $m_p/6\pi a\mu$, relaxation time for momentum transfer
τ_D = $m_p/4\pi a\bar{\rho}D$, relaxation time for mass transfer
τ_T = $m_pc_p/4\pi ak$, relaxation time for droplet temperature
ϕ = $\bar{p}_v/\bar{p}_s$, supersaturation ratio
ω = circular frequency

Superscripts

$\overline{(\ )}$ = mean value over period of acoustic wave
$(\ )'$ = perturbation quantity
$(\ )^*$ = complex conjugate
$\hat{(\ )}$ = peak value

Subscripts

f = final value
I = intensity
i = value at droplet surface
p = droplet
ref = sound reference quantities ($p_{\rm ref} = 2\times10^{-5}$ N/m^2, $I_{\rm ref} = 10^{-12}$ W/m^2)
SPL = sound pressure level
SVL = sound velocity level
s = saturation
v = vapor
0 = value at $x=0$

Presented as Paper 77-1311 at the AIAA 4th Aeroacoustics Conference, Atlanta, Ga., Oct. 3-5, 1977; submitted Oct. 4, 1977; revision received Jan. 17, 1978. Copright © American Institute of Aeronautics and Astronautics, Inc., 1977. All rights reserved.

Index categories: Aeroacoustics; Noise; Multiphase Flows.

*Associate Professor, Dept. of Mechanical and Aerospace Engineering. Member AIAA.

†Graduate Assistant, Dept. of Mechanical and Aerospace Engineering; now Research Assistant, Dept. of Mechanical Engineering, University of Minnesota, Minneapolis.

Introduction

THE attenuation of sound waves by droplets suspended in a gas has been the subject of several theoretical[1-5] and experimental[6,7] investigations. Marble and Candel[8] considered the increased attenuation that would result from injecting droplets into turbofan engine inlets and calculated that the presence of a small concentration (1 to 5% by mass) of micron-sized droplets would result in attenuation of 1-10 dB/m. However, their calculations were based on theory developed for a uniform medium. Addition of droplets to a flowing gas, either by injection or by condensation, will cause temperature and density gradients, and these gradients will affect sound propagation. There is some experimental evidence[9,10] that the attenuation in a condensing vapor in duct or nozzle flows may be greater than predicted by the theory for a uniform gas.

The present work was undertaken to investigate possible reasons for this higher attenuation. A theory was developed to predict the attenuation of a plane acoustic wave propagating in a one-dimensional steady flow of a gas-vapor-droplet mixture. Condensation on the droplets causes gradients in temperature, density, and vapor and droplet concentrations. The theory allows significant gradients in these quantities, provided only that their variation over an acoustic wavelength is not large. In this respect the present analysis differs from the work of Salant and Toong,[11] who assumed that all steady-flow variables changed only slightly over an acoustic wavelength. Also, this paper is concerned with the effect of the droplets, which act as continuously distributed sources of mass, momentum, and energy whose magnitudes depend on the acoustic fluctuations and the gradients in mean flow properties, whereas in Ref. 11 the source terms were left unspecified. It is assumed, however, that in the present case the steady flow is in the low subsonic range and that mean pressure and velocity gradients are small, so that the theory applies to low-speed duct flows rather than the high-speed nozzle flow experiments of Ref. 10.

Theory

Acoustic Equations

The gas-vapor mixture is treated as a perfect gas with constant specific heats, containing sources of mass, momentum, and energy, the rates of addition of these quantities per unit volume being $m, f,$ and q, respectively. The continuity, momentum, and entropy equations for one-dimensional unsteady flow of a compressible gas with sources are linearized in the acoustic velocity and pressure fluctuations u', p', where $u=\bar{u}(x)+u'(x,t)$, $p=\bar{p}(x)+p'(x,t)$. Furthermore, it is assumed that the Mach number $M=\bar{u}/\bar{c}$ of the steady flow is small, and that $\bar{u}$ and $\bar{p}$ vary little over the wavelength λ. The following acoustics equations result:

$$\frac{\partial u'}{\partial t}+\frac{1}{\bar{\rho}}\frac{\partial p'}{\partial x}=\frac{f'}{\bar{\rho}} \tag{1}$$

$$\frac{\partial}{\partial t}\left(\frac{p'}{\gamma\bar{p}}\right)+\frac{\partial u'}{\partial x}=\frac{m'}{\bar{\rho}}+\frac{\gamma-1}{\gamma}\frac{q'}{\bar{p}} \tag{2}$$

The neglected terms are of order M and $M(\lambda/L)$, where L is the characteristic length for variation of $\bar{\rho}$. It has not been assumed that λ/L is small, however; but only that $\lambda/L \ll M^{-1}$.

From Eqs. (1) and (2) one obtains the following equation for the acoustic intensity $I=\overline{p'u'}$:

$$\frac{dI}{dx}=\overline{f'u'}+\frac{1}{\bar{\rho}}\overline{m'p'}+\frac{\gamma-1}{\gamma\bar{p}}\overline{q'p'} \tag{3}$$

Therefore, to this order of approximation, the change in intensity is due entirely to the interaction of the pressure and velocity fluctuations u', p' with the in-phase components of the source terms; there is no energy interchange between the acoustic wave and the mean flow. In the absence of sources the intensity would remain constant, although the sound pressure level would change if there were temperature or density gradients.

Source Terms

The following expressions[8,12,13] are adopted for the source terms:

$$m=4\pi na(\rho D)[(W_v/W)(p_s/p)-r] \tag{4}$$

$$f=6\pi na\mu(u_p-u) \tag{5}$$

$$q=4\pi nak(T_p-T) \tag{6}$$

Linearizing Eq. (4) in the acoustic fluctuations, assuming that $\rho D=$ const and that n and a are unaffected by the sound wave, we obtain for the steady and unsteady parts of the mass source terms

$$\bar{m}/\bar{\rho}=(\bar{r}C_m/\tau_D)[(1/\phi)-1] \tag{7}$$

$$m'/\bar{\rho}=(\bar{r}C_m/\tau_D)\{(1/\phi)[(p_s'/\bar{p}_s)-(p'/\bar{p})]-\sigma'\} \tag{8}$$

Since small particles in a steady flow have negligible slip except in very large gradient fields such as shock waves, we may assume $\bar{u}_p=\bar{u}$, so the steady part of Eq. (5) is zero. The unsteady part is

$$f'/\bar{\rho}=C_m(u_p'-u')/\tau \tag{9}$$

The steady and unsteady parts of the entropy source term, Eq. (6), are, respectively

$$\bar{q}/\bar{\rho}_p c_\ell=(c_p/c_\ell)(\bar{T}_p-\bar{T})/\tau_T \tag{10}$$

$$q'/\rho_p c_\ell=(c_p/c_\ell)(T_p'-T')/\tau_T \tag{11}$$

The source terms involve several variables associated with the vapor and droplet phases. The appropriate equations for these variables are[12]

$$\frac{\partial r}{\partial t}+u\frac{\partial r}{\partial x}=\frac{m}{\rho}(1-r) \tag{12}$$

$$\rho_p\left(\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial x}\right)=-f \tag{13}$$

$$\rho_p c_\ell\left(\frac{\partial T_p}{\partial t}+u_p\frac{\partial T_p}{\partial x}\right)=-mh_L-q \tag{14}$$

$$\frac{dp_s}{dT_p}=h_L p_s/R_v T_p^2 \tag{15}$$

These are linearized and their steady and unsteady parts separated. The steady part of Eq. (12) is

$$\bar{u}\frac{d\bar{r}}{dx}=\bar{m}(1-\bar{r})/\bar{\rho} \tag{16}$$

while the unsteady part may, with the help of Eq. (16), be written

$$\bar{r}\left(\frac{\partial\sigma'}{\partial t}+\bar{u}\frac{\partial\sigma'}{\partial x}\right)+\left(u'+\bar{u}\frac{\rho'}{\bar{\rho}}\right)\frac{d\bar{r}}{dx}=-\frac{\bar{m}}{\bar{\rho}}\sigma'+(1-\bar{r})\frac{m'}{\bar{\rho}} \tag{17}$$

The second and fourth terms on the left of Eq. (17) may be neglected if M and $M(\lambda/L)$ are small. But according to Eq.

(16), the term $u'(\mathrm{d}\bar{r}/\mathrm{d}x)$ is at least of the same order as $(\bar{m}/\bar{\rho})\sigma'$ and must be retained if the latter term is. Equation (17) is simplified and rewritten as

$$\left[\frac{\partial}{\partial t}+\frac{C_m}{\tau_D}\left(\frac{1}{\phi}-1\right)\right]\sigma'-\frac{1-\bar{r}}{\bar{r}}\left(\frac{m'}{\bar{\rho}}\right)=-u'\left(\frac{1}{\bar{r}}\frac{\mathrm{d}\bar{r}}{\mathrm{d}x}\right) \quad (18)$$

Equations (8) and (18) are two coupled equations for σ' and $m'/\bar{\rho}$ in terms of u', p' and p_s'. Linearization of the Clausius-Clapeyron equation [Eq. (15)] gives the following relation between p_s' and T_p':

$$p_s'/\bar{p}_s=H(T_p'/\bar{T}_p) \quad (19)$$

Since $\bar{u}\approx$ const and $\bar{u}\ll\bar{c}$, linearization of Eq. (13) results in

$$\rho_p\frac{\partial u_p'}{\partial t}=-f' \quad (20)$$

This, together with Eq. (9) and the fact that u_p' and u' vary as $\exp(i\omega t)$, gives

$$f'/\bar{\rho}=-i\omega C_m u'/(1+i\omega\tau) \quad (21)$$

for the momentum source term, a purely local relation independent of the gradients.

The steady and unsteady parts of Eq. (14) are, respectively,

$$\bar{u}\frac{\mathrm{d}\bar{T}_p}{\mathrm{d}x}=-(\bar{m}h_L+\bar{q})/\rho_p c_\ell \quad (22)$$

$$\frac{\partial T_p'}{\partial t}=-(m'h_L+q')/\rho_p c_\ell \quad (23)$$

In obtaining Eq. (23) it was assumed that the droplet mean temperature gradient is small. This will be shown subsequently.

Equations are also needed for $\bar{T}$ and T'. These are obtained from the energy equation of the gas-vapor mixture by linearizing and assuming as before that $\bar{p}\approx$ const, and M, $M(\lambda/L)\ll 1$, with the results

$$\bar{u}\frac{\mathrm{d}\bar{T}}{\mathrm{d}x}=\bar{q}/\bar{\rho}c_p \quad (24)$$

$$\frac{\partial}{\partial t}\left(\frac{T'}{\bar{T}}-\frac{\gamma-1}{\gamma}\frac{p'}{\bar{p}}\right)=u'\left(\frac{1}{\bar{\rho}}\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}x}\right)+\frac{\gamma-1}{\gamma}\frac{q'}{\bar{p}} \quad (25)$$

Finally, for the rate of change of the droplet radius we have

$$\bar{u}\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{4\pi}{3}\rho_\ell a^3\right)=-\frac{\bar{m}}{n} \quad (26)$$

Combining Eqs. (7, 10, 16, 22, 24, and 26) leads to a set of four first-order nonlinear ordinary differential equations for $\bar{r}$, $\bar{T}$, $\bar{T}_p$, and a. Solution of these equations requires numerical integration, which is described in the next section. The results provide the x dependence of the coefficients in the acoustics equations, Eqs. (1 and 2). The acoustics equations also contain f', m', q', and so far an explicit result has been obtained only for f', Eq. (21). Expressions for m' and q' are obtained by solving Eqs. (8, 11, 18, 19, 23, and 25). Since the perturbation quantities vary as $\exp(i\omega t)$, these equations reduce to linear algebraic equations, which are solved for m', σ', p_s', q', T_p' and T' in terms of p' and u'. The expressions for m' and q', obtained by straightforward but rather lengthy calculations, are as follows:

$$m'/\bar{\rho}=C_m[\Gamma_1 u'-i\omega\Pi_1(p'/\bar{p})] \quad (27)$$

$$(\gamma-1)q'/\gamma\bar{p}=-C_m[\Gamma_2 u'+i\omega\Pi_2(p'/\bar{p})] \quad (28)$$

where

$$\Gamma_1=\frac{1}{\Delta}\left(i\omega\tau_D\Delta_1\frac{\mathrm{d}\bar{r}}{\mathrm{d}x}+\eta\frac{\bar{r}}{\phi}\Delta_2\frac{\mathrm{d}\ln\bar{\rho}}{\mathrm{d}x}\right) \quad (29a)$$

$$\Gamma_2=\frac{1}{\Delta}\left\{i\omega\tau_D\zeta\theta\frac{\mathrm{d}\bar{r}}{\mathrm{d}x}+\left[\Delta_3+\Delta_2\left(i\omega\tau_D+\zeta H\frac{\bar{r}}{\phi}\right)\right]\frac{\mathrm{d}\ln\bar{\rho}}{\mathrm{d}x}\right\} \quad (29b)$$

$$\Pi_1=\frac{\Delta_2}{\Delta}\left(\Delta_1-\frac{\gamma-1}{\gamma}\eta\right)\frac{\bar{r}}{\phi} \quad (30a)$$

$$\Pi_2=\frac{\gamma-1}{\gamma\Delta}\left\{\Delta_3+\Delta_2\left[i\omega\tau_D+\left(H-\frac{\gamma}{\gamma-1}\theta\right)\zeta\frac{\bar{r}}{\phi}\right]\right\} \quad (30b)$$

$$\Delta=\Delta_1 i\omega\tau_D\left[i\omega\tau_D+C_m\left(\frac{1}{\phi}-\bar{r}\right)\right]+\Delta_2(i\omega\tau_T+C_m)\zeta H\frac{\bar{r}}{\phi} \quad (30c)$$

$$\Delta_1=i\omega\tau_T+C_m+(c_p/c_\ell) \quad (30d)$$

$$\Delta_2=i\omega\tau_D+C_m\left(\frac{1}{\phi}-1\right) \quad (30e)$$

$$\Delta_3=i\omega\tau_D(1-\bar{r})C_m \quad (30f)$$

Equations (27) and (28) contain gradients of the gas density and vapor mass fraction in the coefficients $\Gamma_{1,2}$. These equations may be reduced to the uniform-gas results of Marble and Candel[8] by dropping the latter terms, taking $\theta=\phi=1$, and making further approximations for small C_m. It should be noted, however, that if the droplets are not in equilibrium with the vapor, $\phi\neq 1$, and the Marble-Candel approximation is obtained only by making the additional assumption that $\omega\tau_D/C_m\gg 1$.

Numerical Solutions and Results

Solution for Mean Quantities

The system of ordinary differential equations for $\bar{r}$, $\bar{T}$, $\bar{T}_p$, and a are nondimensionalized by introducing the characteristic length for variation of droplet mass

$$L=\bar{u}\tau_{D0}/C_{m0}$$

and defining the dimensionless variables $\xi=x/L$ and

$$y_1=\bar{r}/\bar{r}_0,\quad y_2=\bar{T}/\bar{T}_0,\quad y_3=\bar{T}_p/\bar{T}_{p0},\quad y_4=a/a_0$$

Since $\bar{p}$ is assumed constant, $\bar{\rho}/\bar{\rho}_0=1/y_2$. Equations (7, 10, 16, 22, 24, and 26) may be written as follows in terms of these variables:

$$\frac{\mathrm{d}y_1}{\mathrm{d}\xi}=y_1y_2y_4(1-y_1\bar{r}_0)\left(\frac{1}{\phi}-1\right) \quad (31)$$

$$\frac{\mathrm{d}y_2}{\mathrm{d}\xi}=N_{Le}y_2y_4(y_3\theta_0-y_2) \quad (32)$$

$$\frac{\mathrm{d}y_3}{\mathrm{d}\xi}=-\frac{(c_p/c_\ell)}{C_{m0}y_4^2}\left[N_{Le}\left(y_3-\frac{y_2}{\theta_0}\right)+\frac{h_L\bar{r}_0}{c_p\bar{T}_{p0}}y_1\left(\frac{1}{\phi}-1\right)\right] \quad (33)$$

$$\frac{\mathrm{d}y_4}{\mathrm{d}\xi}=-\frac{1}{3}\frac{\bar{r}_0}{C_{m0}}\frac{y_1}{y_4}\left(\frac{1}{\phi}-1\right) \quad (34)$$

The Lewis number is assumed constant. The supersaturation ratio ϕ is a function of y_1 and y_3, since

$$\phi = (W/W_v)(\bar{p}/\bar{p}_s)\bar{r}_0 y_1$$

and $\bar{p}_s$, obtained by integrating the Clausius-Clapeyron equation for constant h_L, is

$$\bar{p}_s(\xi) = \bar{p}_s(0)\exp\left[H_0\left(1 - \frac{1}{y_3}\right)\right]$$

Equations (31-34) were solved numerically by the fourth-order Runge-Kutta method for the initial conditions $y_j(0) = 1$ ($j = 1, \ldots, 4$). Typical results for an air-water vapor mixture containing droplets are shown by the solid curves in Fig. 1. The initial vapor mass fraction $\bar{r}_0$ was taken as 0.05, which is larger than the saturation value of 0.0174 corresponding to the assumed initial temperature of the air-water vapor mixture (23.1°C) and 1 atm. The initial temperatures of the droplets and the air-vapor mixture were assumed to be equal in this case. At first the droplet temperature increases rapidly as the influx of latent heat from the condensing vapor is utilized to raise the vapor pressure of the liquid in an attempt to bring it into equilibrium with the vapor. This occurs in a time of order τ_T, or distance $\xi \sim O(C_m)$. Thereafter the droplet temperature remains essentially constant as energy released in condensation is transferred from the droplets to the gas-vapor mixture, causing the temperature of the latter to rise. This process requires a much longer time, of order τ_D/C_m, or distance $\xi \sim O(1)$.

The numerical results indicate that the temperature to which the droplets rapidly adjust is slightly above the saturation temperature corresponding to the initial vapor mass fraction. Since the change in droplet temperature depends on the initial temperature difference between the droplets and gas, which is in reality unknown, it is not of great importance in the present problem. Considerable simplification results if the droplet temperature is assumed constant and equal to its final value, which is determined by setting $dy_3/d\xi = 0$ in Eq. (33):

$$y_3 = \frac{1}{\theta_0}\left[y_2 - \frac{h_L \bar{r}_0}{c_p \bar{T}_0 N_{Le}} y_1 \left(\frac{1}{\phi} - 1\right)\right] \tag{35}$$

Since y_1 and y_2 do not differ much from their initial values when y_3 has completed its rapid adjustment, a close approximation to the final droplet temperature can be obtained from this equation by taking $y_1 = y_2 = 1$. Equation (35) is solved for y_3 by iteration because ϕ depends on y_3. From Eqs. (31, 32, and 35) we obtain

$$y_2 = 1 + \frac{h_L}{c_p \bar{T}_0} \ln\left(\frac{1 - y_1 \bar{r}_0}{1 - \bar{r}_0}\right) \tag{36}$$

which provides a simple relation between the temperature rise of the gas and the vapor mass fraction. The following relation between the droplet radius and gas temperature is obtained from Eqs. (32, 34, 35):

$$y_4 = \left(1 + \frac{c_p \bar{T}_0}{h_L C_{m0}} \ln y_2\right)^{1/3} \tag{37}$$

Substitution of Eq. (37) into Eq. (32) leads to a single first-order differential equation in y_2 which is solved numerically. The dashed curves in Fig. 1 correspond to the solutions obtained in this manner. The approximate results are in excellent agreement with the exact solutions of Eqs. (31-34); in fact, the curves for the gas temperature and density are coincident. Therefore, the simpler approximate method was used to calculate the mean quantities needed for solution of the acoustics equations. These mean quantities are replotted as functions of the actual distance x in Fig. 2 for $\bar{u} = 100$ m/s and

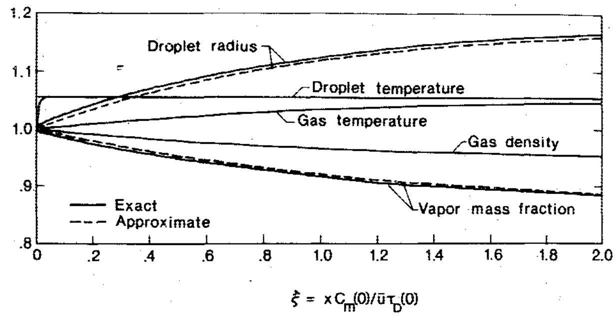

Fig. 1 Conditions in nonuniform medium due to condensation of an initially supersaturated air-water vapor mixture. All mean quantities are normalized by their values at $x = 0$. Initial conditions $C_{m0} = 0.01$, $\bar{r}_0 = 0.05$, $\bar{T}_{p0} = \bar{T}_0 = 296.3$ K.

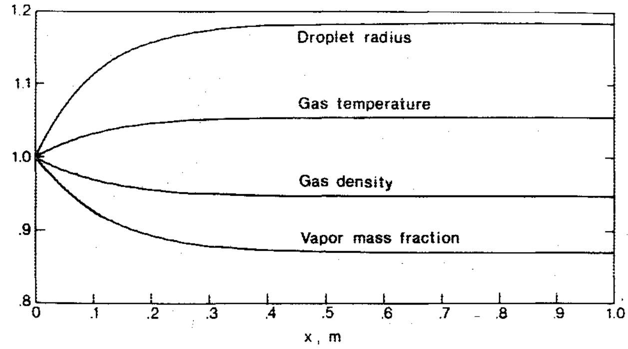

Fig. 2 Variation of mean quantities with actual distance x. Other conditions as Fig. 1; initial droplet radius $a_0 = 1$ μm; mean flow velocity $\bar{u} = 100$ m/s.

$a_0 = 1$ μm. In this case their gradients are significant within about the first 0.3 m. Since $\bar{u}$ appears only as a proportionality constant in the characteristic length L, results for other $\bar{u}$ are easily obtained by rescaling the abscissa.

Solution of Acoustic Equations

The acoustic variables are written

$$p'(x,t) = \hat{p}_0 P(x)\exp(i\omega t) \tag{38a}$$

$$u'(x,t) = (\hat{p}_0/\bar{\rho}_0\bar{c}_0)\, U(x)\exp(i\omega t) \tag{38b}$$

where $\hat{p}_0$ is the amplitude of the sound pressure at $x = 0$. Substituting Eqs. (21, 27, 28, and 38) into the acoustics equations [Eqs. (1) and (2)], we obtain the following two linear ordinary differential equations for P and U:

$$y_2 \frac{dP}{dx} + i\kappa_0\left(1 + \frac{C_m}{1 + i\omega\tau}\right)U = 0 \tag{39}$$

$$\frac{dU}{dx} - C_m(\Gamma_1 - \Gamma_2)U + i\kappa_0[1 + \gamma C_m(\Pi_1 + \Pi_2)]P = 0 \tag{40}$$

where $\kappa_0 = \omega/\bar{c}_0$. The quantities y_2, C_m, τ, $\Gamma_{1,2}$, and $\Pi_{1,2}$ are known functions of x which are evaluated from the solutions for the mean quantities.‡ It should be noted that Γ_1 and Γ_2 depend on the gradients of $\bar{\rho}$ and $\bar{r}$. Equations (39) and (40) are solved numerically using the fourth-order Runge-Kutta technique. The boundary conditions require special consideration. Although by definition $P(0) = 1$, the value of $U(0)$ is unknown, except in the trivial case of a uniform medium without droplets, when it is 1. The other boundary condition

‡Incidentally, the sound speed $\bar{c}$ is also a function of x, varying as $y_2^{1/2}$.

must be applied at infinity, where the conditions in the medium are no longer changing. The appropriate condition at infinity is that the impedance $Z=p'/u'$ approaches the constant (complex) value appropriate for wave propagation in a uniform medium. This value of Z, which will be denoted Z_f, is obtained from Eqs. (39) and (40) by setting the coefficients equal to their final values, taking $P,\ U\propto\exp(-i\kappa x)$, and solving the resulting homogeneous algebraic equations. The final values of the mean quantities are actually evaluated at a finite distance x_f from the origin, so the boundary conditions are

$$P(0)=1 \tag{41}$$

$$U(x_f)=(\rho_0 c_0/Z_f)P(x_f) \tag{42}$$

Numerical integration of Eqs. (39) and (40) starting from $x=0$ with an assumed value of $U(0)$ sometimes encountered instabilities apparently associated with the incorrect starting value. It was found that instabilities could be avoided by backwards integration from $x=x_f$ to $x=0$. The integration is carried out for a trial value of $P(x_f)$. Since both starting conditions at x_f are proportional to this value, and Eqs. (39) and (40) are linear, the solutions are also proportional to the assumed $P(x_f)$. Therefore, the solution satisfying Eq. (41) can be obtained by simply dividing the results calculated for the trial $P(x_f)$ by the corresponding value of $P(0)$. The numerical solutions were checked against analytical solutions available for the special cases of a uniform medium with and without droplets and were found to be in excellent agreement.

Figure 3 shows the x variation of the acoustic pressure and velocity fluctuations for a wave propagating in the same direction as the steady flow of the nonuniform medium whose properties are given in Figs. 1 and 2. The curves in Fig. 3 represent the real and imaginary parts of $P(x)$ and $U(x)$ normalized by their values at $x=0$. The corresponding amplitudes are shown in Fig. 4, along with the normalized intensity $I(x)/I(0)$, where I is calculated from the numerical results using the relation

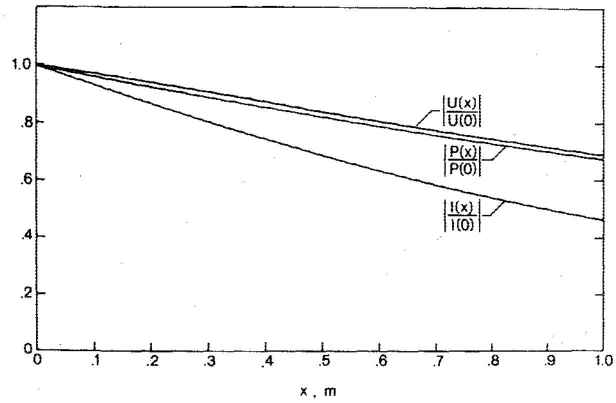

Fig. 4 Attenuation of acoustic pressure and velocity amplitudes and intensity. Same conditions as Figs. 1-3.

$$I=\overline{p'u'}=(\hat{p}_0^2/4\rho_0 c_0)\,(PU^*+P^*U) \tag{43}$$

The pressure, velocity, and intensity are all attenuated, but unlike the case for a uniform medium, the attenuation of pressure and velocity are not quite the same.

The difference in attenuation is seen more clearly in terms of the local attenuation coefficients (dB/m) for intensity, sound pressure, and velocity§

$$\alpha_I=-\frac{dL_I}{dx} \tag{44a}$$

$$\alpha_{SPL}=-\frac{dL_P}{dx} \tag{44b}$$

$$\alpha_{SVL}=-\frac{dL_V}{dx} \tag{44c}$$

For a uniform dissipative medium these coefficients are positive constants, and $\alpha_I=\alpha_{SPL}=\alpha_{SVL}$. Figure 5 shows their variation with x in the condensing vapor for the same conditions as in Figs. 1-4. The upper set of curves is for the nonuniform gas-vapor mixture with droplets. The lower set of curves shows the attenuation coefficients in the same nonuniform gas without droplets so that the effect of the temperature and density gradients alone can be seen. In both cases α_{SPL} is larger and α_{SVL} is smaller than α_I in the region where the temperature and density gradients are important. The difference is not insignificant, since it amounts to about 1 dB/m at $x=0$. Since the sound pressure level is the quantity usually measured in experiments, the fact that α_{SPL} is larger than α_I is interesting. Another interesting result is that α_{SPL} is nearly constant throughout the gradient region in the droplet-laden gas. Also, comparison of the upper and lower sets of curves indicates that the difference between α_{SPL}, α_I, and α_{SVL} is due to the gradients, which affect the sound pressure and velocity more than the intensity. The attenuation in intensity is due mainly to dissipation, gradients having relatively little effect. In the absence of droplets, $\alpha_I=0$, whereas in the condensing droplet-laden gas, α_I increases along the flow as the droplet mass fraction C_m increases, in this case from 0.01

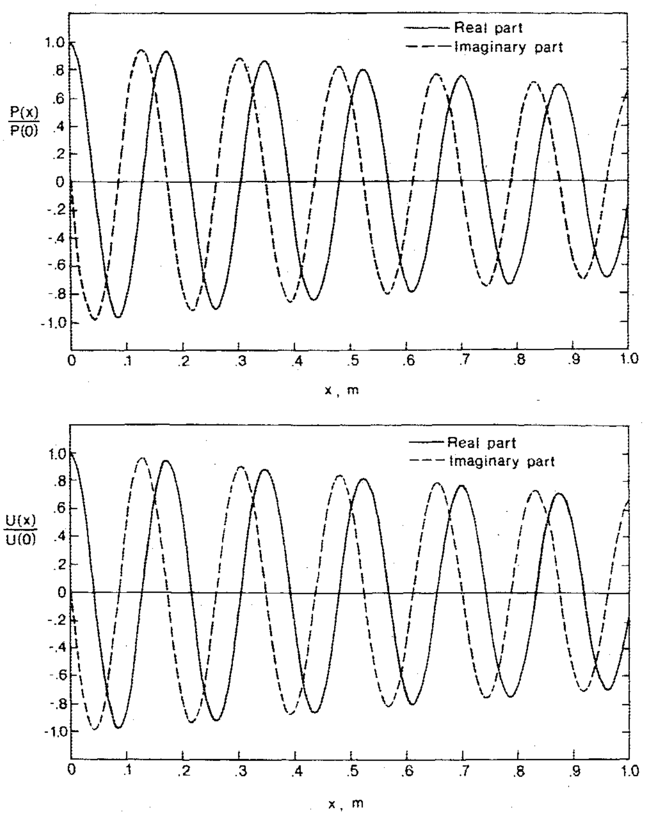

Fig. 3 Spatial variation of acoustic pressure and velocity in nonuniform air-water vapor mixture containing water droplets of initial radius 1 μm. Frequency 2 kHz; other conditions as in Figs. 1-2.

§Only α_{SPL} is in common usage. The intensity attenuation coefficient, which is similar to that defined in Ref. 11, is convenient because it refers to the decrease in acoustic energy flux. Sometimes an energy attenuation coefficient α_E is defined by considering the change in the acoustic energy density $E=(\tfrac{1}{2})[(p'^2/\bar{\rho}\bar{c}^2)+\bar{\rho}u'^2]$, but this is not useful here. In general $\alpha_E\neq\alpha_I$, for even when the relation $u'=p'/\bar{\rho}\bar{c}$ is approximately satisfied, so that $I=E\bar{c}$, the variation of the sound speed causes them to differ.

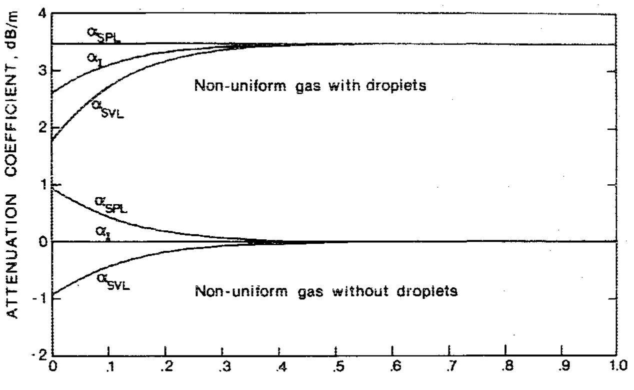

Fig. 5 Attenuation coefficients for sound pressure level, intensity, and sound velocity level. Same conditions as Figs. 1-4.

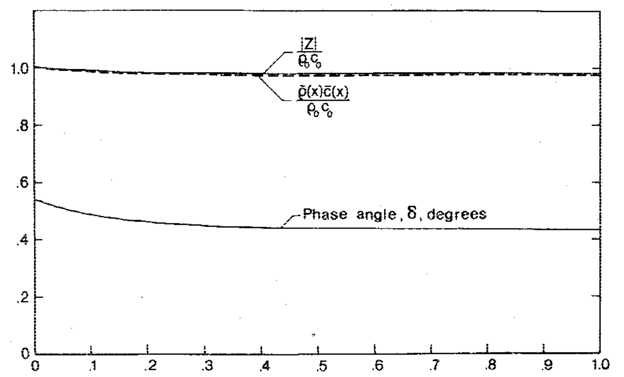

Fig. 6 Modulus and phase angle of impedance. Same conditions as Figs. 1-5.

to 0.0175. This increase in C_m more than offsets the tendency of α_I to decrease with increasing droplet size.

The following result, which exhibits more clearly the dependence of the various attenuation coefficients on the gradients, is obtained from Eqs. (3, 21, 27, and 28):

$$\alpha_I = 4.343 C_m \{ -R_e[(\Gamma_1 - \Gamma_2)Z^*] + \bar\rho[\omega^2\tau/(1+\omega^2\tau^2)] - (\omega/\bar\rho) Im(\Pi_1 + \Pi_2)\,|Z|^2\}[Re(Z)]^{-1} \tag{45}$$

Gradients appear explicitly only through the quantities $\Gamma_{1,2}$ in the first term of this equation. This term is, however, usually smaller than the other terms. In fact, for the conditions of Figs. 1-5, it was found to be completely negligible. It has a perceptible effect only at low frequencies and low mean velocities. For example, at a frequency of 500 Hz and $\bar u = 30$ m/s, other conditions being the same, its effect is to decrease α_I by about 0.3 dB/m. If this term is neglected, the only remaining effect of gradients is on the impedance Z, which must be obtained by solving Eqs. (39) and (40).

The amplitude and phase of the impedance are plotted as functions of x in Fig. 6, from which it is clear that the impedance is approximately equal to $\bar\rho(x)\bar c(x)$. With this approximation and neglect of the gradient terms, Eq. (45) becomes the same as the expression that would be obtained by assuming a quasiuniform medium and evaluating $\alpha_I(x)$ for the local conditions prevailing at each value of x. No such local relations can be derived for the attenuation coefficients for sound pressure and velocity, because the gradients directly affect the quantities through their influence on the impedance. In fact, eliminating U from Eq. (43) by introducing the impedance $Z = |Z|\exp(i\delta)$ and using Eqs. (44a, b), we obtain the following relation between α_{SPL} and α_I:

$$\alpha_{SPL} = \alpha_I - 4.343\left(\frac{1}{|Z|}\frac{d|Z|}{dx} + \tan\delta\frac{d\delta}{dx}\right) \tag{46}$$

Since δ is small, most of the difference between α_{SPL} and α_I is due to the x variation of $|Z|$ caused by the axial temperature gradient. Thus, for small C_m and small gradients reasonably accurate results may be obtained by calculating α_I from Eq. (45) neglecting the $\Gamma_1 - \Gamma_2$ term and taking $Z = \bar\rho(x)\bar c(x)$, and then determining α_{SPL} from Eq. (46) assuming $\delta = 0$. The validity of this approximation of course depends on the smallness of the gradient term compared to the other terms in Eq. (45), which in turn requires that $\lambda/L \ll 1$. When this condition is not satisfied it is necessary to use the full theory for accurate results.

Conclusions

A theory has been developed to predict acoustic attenuation in a flowing gas-vapor mixture in which condensation onto suspended droplets causes gradients in temperature and density along the direction of flow. The theory shows that these gradients affect acoustic attenuation in two ways. First, they cause changes in the acoustic impedance of the medium which affect the sound pressure level but have relatively little effect on the sound intensity. Second, gradients influence the mass and heat transfer between the droplets and the gas-vapor mixture and therefore alter the dissipative processes which are responsible for attenuation of sound intensity. As a result, the attenuation of sound pressure level and intensity are not the same in a nonuniform medium. In a condensing vapor the attenuation of sound pressure level is greater than that of intensity. Calculations show that the corresponding attenuation coefficients differ by almost 1 dB/m in the region where the gradients are largest, but the difference rapidly diminishes as the gas-vapor mixture approaches uniform conditions, so the overall difference in attenuation is smaller. The effect of gradients on attenuation of sound pressure level is mainly due to the change in impedance. Their direct effect on the dissipative process is small except at low frequencies, when the acoustic wavelength is of the same order as the characteristic distance for variation of the gas density and temperature.

Acknowledgment

The work was supported by the National Science Foundation under Grant ENG73-04257.

References

[1] Epstein, P. S. and Carhart, R. R., "The Absorption of Sound in Suspensions and Emulsions. I. Water Fog in Air," *Journal of the Acoustical Society of America*, Vol. 25, May 1953, pp. 553-565.

[2] Temkin, S. and Dobbins, R. A., "Attenuation and Dispersion of Sound by Particulate-Relaxation Processes," *Journal of the Acoustical Society of America*, Vol. 40, Aug. 1966, pp. 317-324.

[3] Marble, F. E. and Wooten, D. C., "Sound Attenuation in a Condensing Vapor," *Physics of Fluids*, Vol. 13, May 1970, pp. 2657-2664.

[4] Cole, J. E., III and Dobbins, R. A., "Propagation of Sound Through Atmospheric Fog," *Journal of the Atmospheric Sciences*, Vol. 27, May 1970, pp. 426-434.

[5] Davidson, G. A., "Sound Propagation in Fogs," *Journal of the Atmospheric Sciences*, Vol. 32, Nov. 1975, pp. 2201-2205.

[6] Temkin, S. and Dobbins, R. A., "Measurements of Attenuation and Dispersion of Sound by an Aerosol," *Journal of the Acoustical Society of America*, Vol. 40, Nov. 1966, pp. 1016-1024.

[7] Cole, J. E., III and Dobbins, R. A., "Measurements of the Attenuation of Sound by a Warm Air Fog," *Journal of the Atmospheric Sciences*, Vol. 28, March 1971, pp. 202-209.

[8] Marble, F. E. and Candel, S. M., "Acoustic Attenuation in Fans and Ducts by Vaporization of Liquid Droplets," *AIAA Journal*, Vol. 13, May 1975, pp. 634-639.

[9]Bushell, K. W., private communication, 1974, regarding unpublished data of W. F. Ball, Rolls-Royce Ltd., 1971.

[10]Jaeschke, M., Hiller, W. J., and Meier, G. E. A., "Acoustic Damping in a Gas Mixture with Suspended Submicroscopic Droplets," *Journal of Sound and Vibration*, Vol. 43, Dec. 1975, pp. 467-481.

[11]Salant, R. F. and Toong, T.-Y., "Amplification and Attenuation of Acoustic Waves in Nonhomogeneous Steady Flows," *Journal of the Acoustical Society of America*, Vol. 49, May 1971, pp. 1655-1666.

[12]Marble, F. E., "Some Gasdynamic Problems in the Flow of Condensing Vapors," *Astronautica Acta*, Vol. 14, Oct. 1969, pp. 585-614.

[13]Marble, F. E., "Acoustic Attenuation by Vaporization of Liquid Droplets—Application to Noise Reduction in Aircraft Powerplants," California Institute of Technology, Pasadena, Calif., Rept. AFOSR-TR-0511, Feb. 1975.

From the AIAA Progress in Astronautics and Aeronautics Series . . .

SATELLITE COMMUNICATIONS:
FUTURE SYSTEMS-v. 54
ADVANCED TECHNOLOGIES-v. 55

Edited by David Jarett, TRW, Inc.

Volume 54 and its companion Volume 55, provide a comprehensive treatment of the satellite communication systems that are expected to be operational in the 1980's and of the technologies that will make these new systems possible. Cost effectiveness is emphasized in each volume, along with the technical content.

Volume 54 on future systems contains authoritative papers on future communication satellite systems in each of the following four classes: North American Domestic Systems, Intelsat Systems, National and Regional Systems, and Defense Systems. **A significant part of the material has never been published before.** Volume 54 also contains a comprehensive chapter on launch vehicles and facilities, from present-day expendable launch vehicles through the still developing Space Shuttle and the Intermediate Upper Stage, and on to alternative space transportation systems for geostationary payloads. All of these present options and choices for the communications satellite engineer. The last chapter in Volume 54 contains a number of papers dealing with advanced system concepts, again treating topics either not previously published or extensions of previously published works.

Volume 55 on advanced technologies presents a series of new and relevant papers on advanced spacecraft engineering mechanics, representing advances in the state of the art. It includes new and improved spacecraft attitude control subsystems, spacecraft electrical power, propulsion subsystems, spacecraft antennas, spacecraft RF subsystems, and new earth station technologies. Other topics are the relatively unappreciated effects of high-frequency wind gusts on earth station antenna tracking performance, multiple-beam antennas for higher frequency bands, and automatic compensation of cross-polarization coupling in satellite communication systems.

With the exception of the first "visionary" paper in Volume 54, all of these papers were selected from the 1976 AIAA/CASI 6th Communication Satellite Systems Conference held in Montreal, Canada, in April 1976, and were revised and updated to fit the theme of communication satellites for the 1980's. These archive volumes should form a valuable addition to a communication engineer's active library.

Volume 54, 541 pp., 6×9, illus., $19.00 Mem., $35.00 List
Volume 55, 489 pp., 6×9, illus., $19.00 Mem., $35.00 List
Two-Volume Set (Vols. 54 and 55), $55.00 Mem. & List

TO ORDER WRITE: Publications Dept., AIAA, 1290 Avenue of the Americas, New York, N. Y. 10019